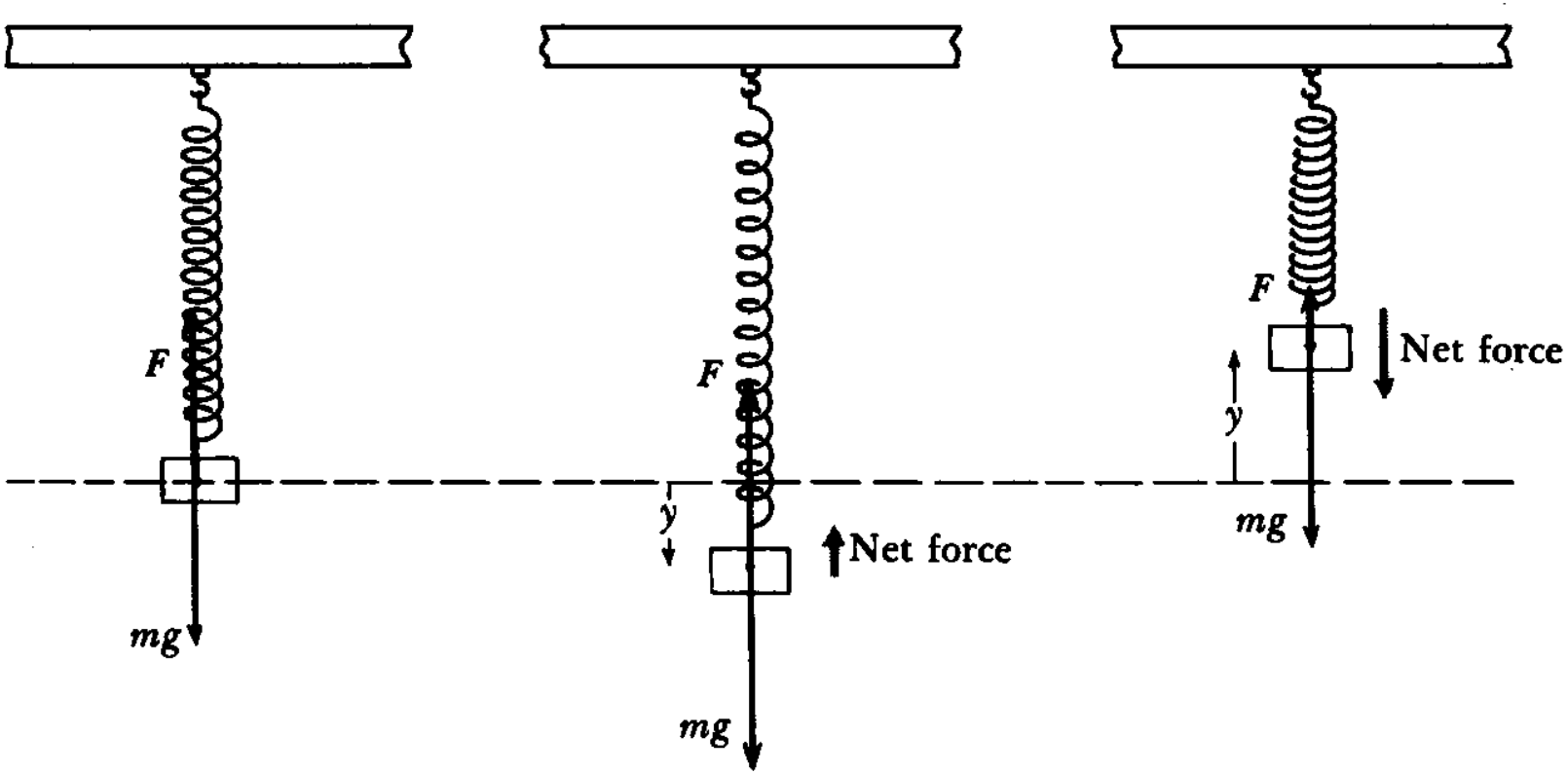


Hooke's law - F = - ky

Simple Harmonic Motion (SHM)

$F = ma$ & $F = -kx$

position: $x = A \cos \omega t$

A: amplitude

velocity: $v = -A\omega \sin \omega t$

$v_{max} = A\omega$: maximum velocity

$\omega = (k/m)^{1/2}$ - angular frequency

$f = \omega / 2\pi$ - frequency

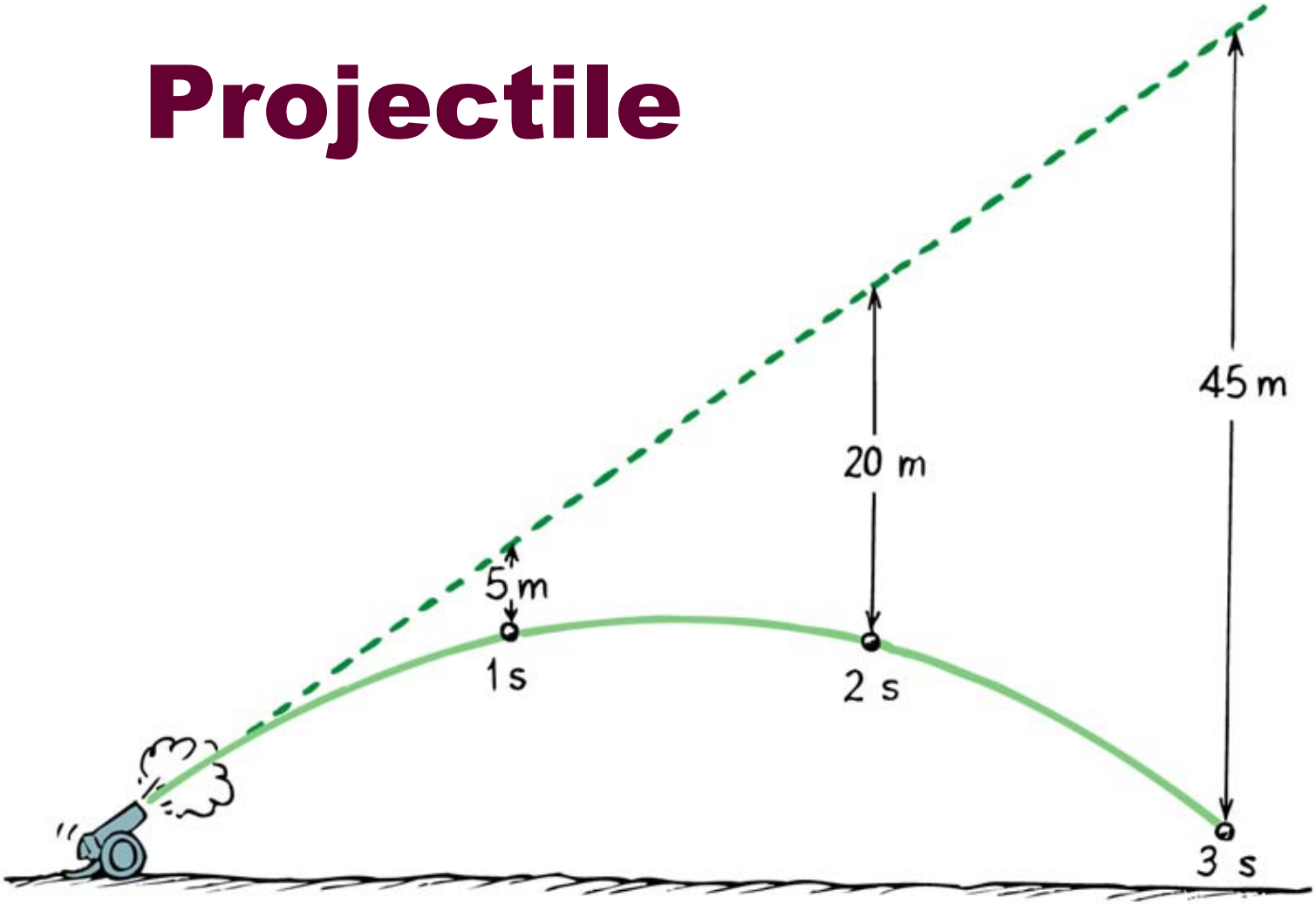
$\tau = 1/f = 2\pi / \omega$ - period

$PE = kx^2/2$ & $KE = mv^2/2$

Conservation of energy

PE+KE=constant

Projectile



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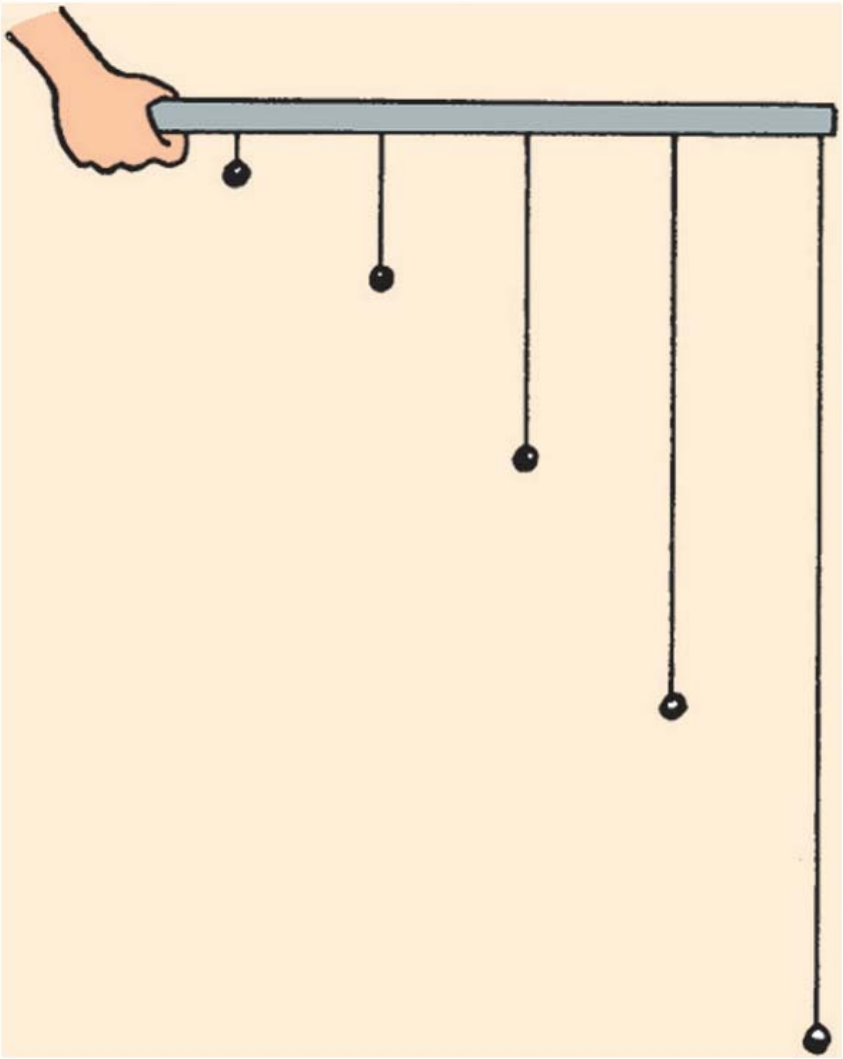

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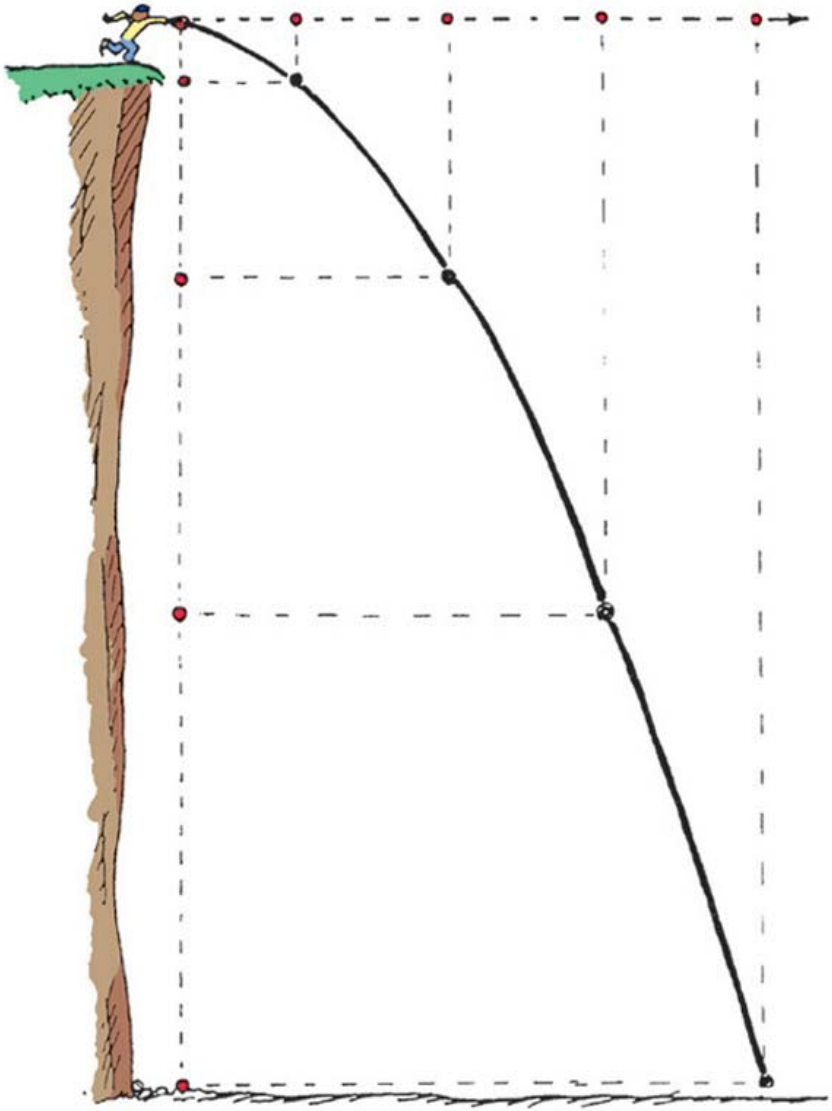

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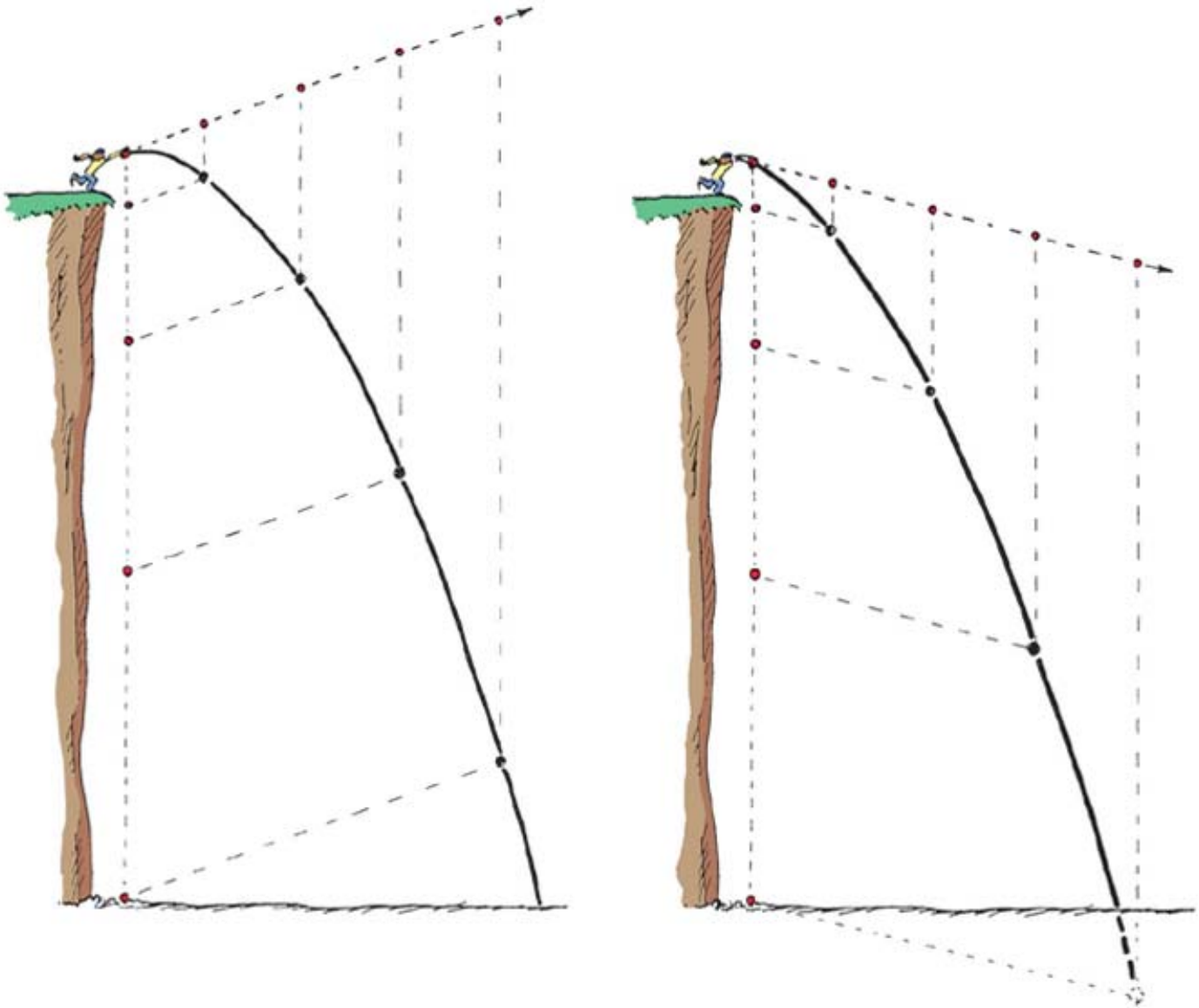

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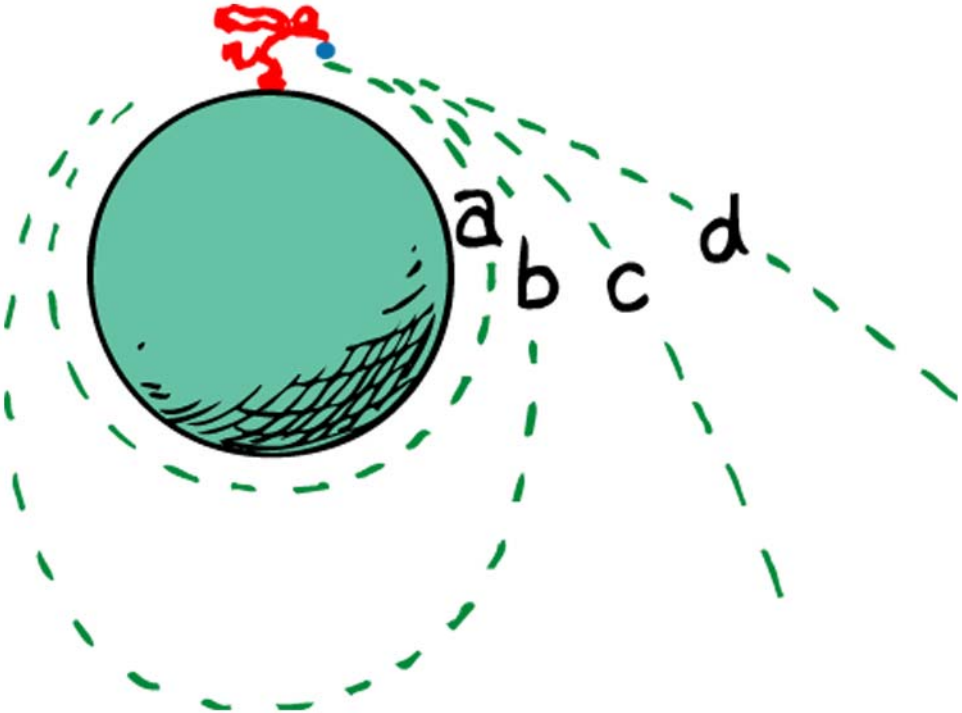

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Planetary motion

Kepler's first law

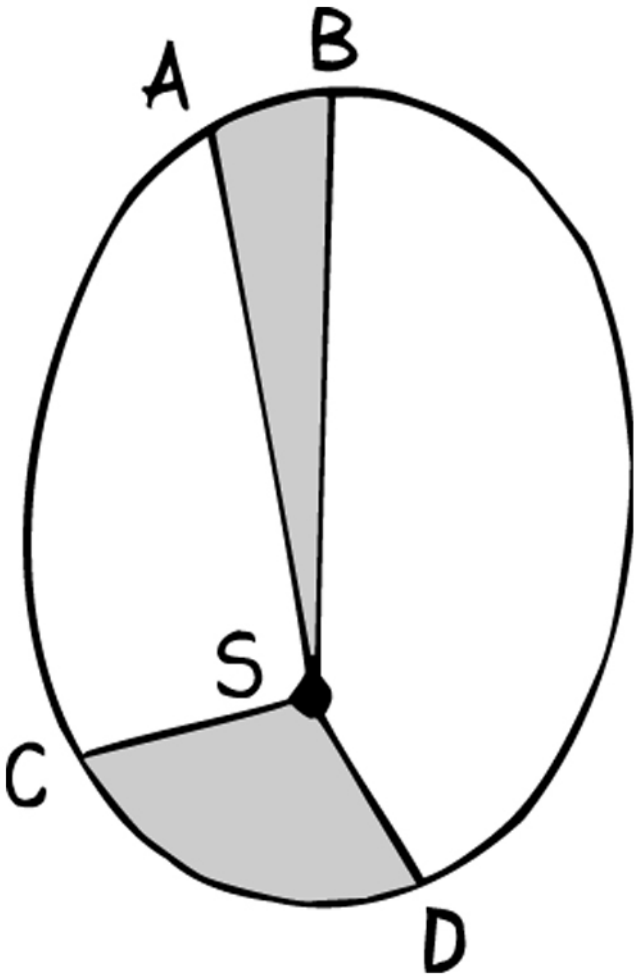
The path of each planet around the sun is an ellipse with the sun at one of its focus

Kepler's third law

The square of the period τ of a planetary orbital motion is proportional to the cube of the average distance (i.e., r) of the planet from the sun, i.e.,

$$\tau \propto r^{3/2}$$

Conservation of Angular Momentum



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Kepler's second law

Equal areas in equal time

Centripetal acceleration

$$a = v^2/r$$

Gravitational force due the earth is the source of the centripetal force which keeps moon in its orbit, i.e.,

$$GMm/r^2 = mv^2/r, \text{ or}$$

$$v = (GM/r)^{1/2}$$

is the velocity of the moon moving in a nearly circular orbit around the earth.

Since the period

$$\tau = 2\pi r/v = 2\pi r/(GM/r)^{1/2}$$

and

$$\tau^2 = (4\pi^2/GM)\, r^3$$

(Kepler's 3rd law)

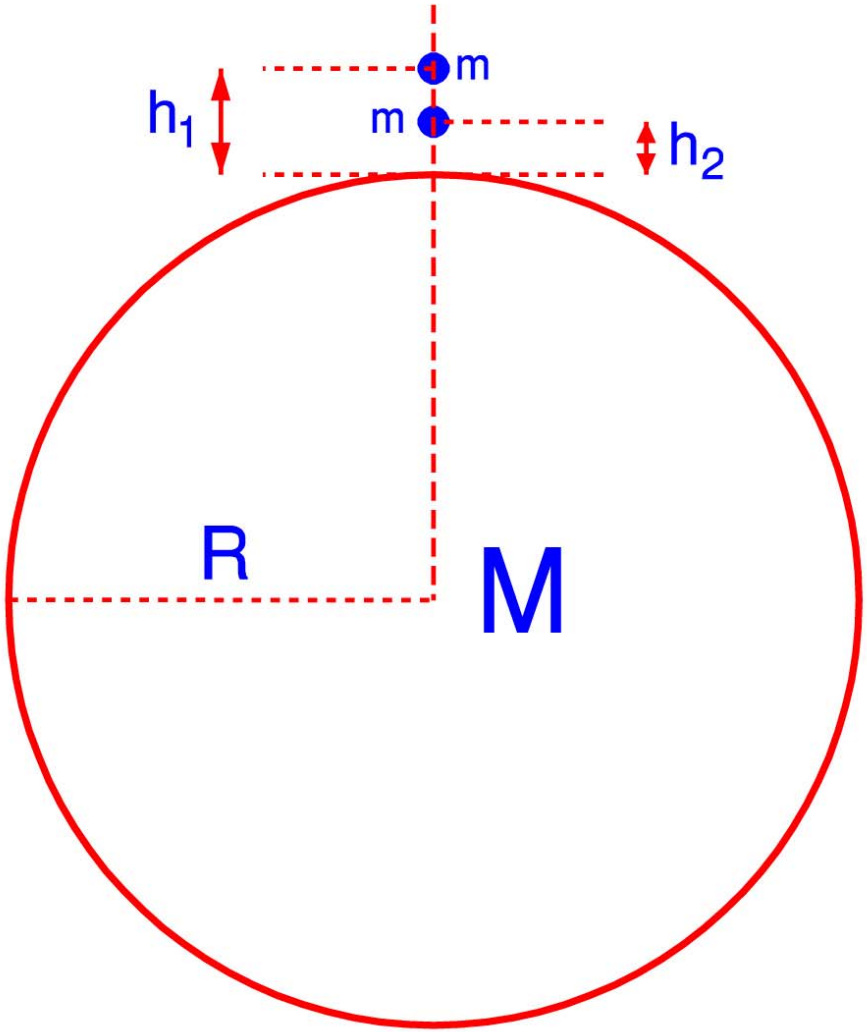
Satellite is an object with mass m which orbits around another massive object with mass M at an average distance r apart. In general, the satellite orbits in an elliptical orbit (or, a nearly circular orbit) at a speed $v = (GM/r)^{1/2}$ and its total energy, i.e., KE+PE, is conserved.

A geo-synchronous satellite orbits the earth with the same orbital period as the rotating earth. Observing from the earth, it is on a fixed spot above and does not appear to move in the sky. Its geo-stationary location in space (at an altitude of 36,000 km above the sea level) supports the modern telecommunication networks.

PE due to gravitational force

$$PE = - GMm / r$$

Is this expression consistent with the earlier definition of PE, i.e., $PE = mgh$?



$$PE = -GMm / r$$

$$r_1 = R + h_1$$
$$r_2 = R + h_2$$

$$PE_1 - PE_2 = ?$$

$$1 / (1+x) = 1 - x, \quad \text{when } x \ll 1$$

$$1 / (A+a) = 1/(A(1+a/A)) = (1-a/A)/A$$

$$= (A-a)/A^2, \quad \text{when } a \ll A$$

$$R \gg h_1 \ \& \ h_2$$

$$PE_1 = - GMm/(R+h_1) = - (GMm/R^2)(R-h_1)$$

$$= - m\,(GM/R^2)\,(R-h_1) = - mg\,(R-h_1)$$

$$PE_2 = - GMm/(R+h_2) = - (GMm/R^2)(R-h_2)$$

$$= - m\,(GM/R^2)\,(R-h_2) = - mg\,(R-h_2)$$

$$PE_1 - PE_2 = mg\,[(R-h_2) - (R-h_1)]$$

$$= mg\,(h_1 - h_2)$$

Escape velocity

Conservation of energy

$$mv_1^2/2 - GMm/r_1 = mv_2^2/2 - GMm/r_2$$

$v_2 = 0$ at $r_2 \rightarrow \infty$ & $v_1 = v_{es}$ at $r_1 = R_{earth}$

$$mv_{es}^2/2 - GM_e m/R_e = 0$$

$v_{es}^2 = 2GM_e/R_e$ or $v_{es} = (2GM_e/R_e)^{1/2}$

v_{es} (earth) = 11.2 km/sec

Chapter 9, example #1

What would be the value of g (i.e., the constant acceleration due to gravity) near the surface of a planet of radius half of the radius of the earth and a mass 5 times the mass of the earth? What would be the weight of an object of mass m = 50 kg near the surface of this planet?

Ans: 200 m/sec^2 and 10,000 N.

Chapter 9, example #2

How large is the centripetal acceleration for an object of mass 2 kg moving in a circular path of radius r = 0.4 m with a constant speed v = 4 m/sec? How large is the centripetal force that keeps this object moving in the circular path?

Ans: 40 m/sec^2 and 80 N.

Chapter 9, example #3

What would be the g value inside the earth at a distance 1/4 of the earth radius from the center of the earth? What would be the weight of an object of mass 4 kg at that distance from the center of the earth? How much would this object weigh at the surface of the earth?

Ans: 2.5 m/sec^2, 10 N, and 40 N.